Minitest 4A - MTH 1410

Dr. Graham-Squire, Spring 2014

10:52
11:03
11 min.

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

(*signature*)

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation. A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are not allowed on this test. Calculators are allowed on the first — questions of the test, however you should still show all of your work. No calculators are allowed on the last — questions of the test.
5. You will be able to come back to the calculator portion of the test, but you cannot come back to the No Calculator portion after you turn it in.
6. If you need it, the quadratic formula is $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
8. Make sure you sign the pledge.
9. Number of questions = 5. Total Points = 30.

1. (6 points) Fill in the blanks to use the limit definition of definite integral to calculate $\int_0^2 (3x^3-7)\,dx$. The calculation is done with respect to right endpoints.

$\frac{2}{n}$

0 2

$$\int_0^2 (3x^3-7)\,dx$$

$$=\lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x$$

$$=\lim_{n\to\infty}\frac{2}{n}\left(f\left(\frac{2}{n}\right)+f\left(\frac{4}{n}\right)+f\left(\frac{6}{n}\right)+\cdots+f\left(\frac{2n}{n}\right)\right)$$

$$=\lim_{n\to\infty}\frac{2}{n}\left[\left(3\left(\frac{2}{n}\right)^3-7\right)+\left(3\left(\frac{4}{n}\right)^3-7\right)+\left(3\left(\frac{6}{n}\right)^3-7\right)+\cdots+\left(3\left(\frac{2n}{n}\right)^3-7\right)\right]$$

$$=\lim_{n\to\infty}\frac{2}{n}\left[\left(3\left(\frac{2}{n}\right)^3+3\left(\frac{4}{n}\right)^3+3\left(\frac{6}{n}\right)^3+\cdots+3\left(\frac{2n}{n}\right)^3\right)-(7+7+7+\cdots+7)\right]$$

more space

$$=\lim_{n\to\infty}\frac{2}{n}\left[3\left(\frac{2}{n}\right)^3(1^3+2^3+3^3+\cdots+n^3)-7(n)\right]$$

$$=\lim_{n\to\infty}\frac{2}{n}\left[3\left(\frac{8}{n^3}\right)\left(\frac{n(n+1)}{2}\right)^2-7(n)\right]$$

$$=\lim_{n\to\infty}\frac{\cancel{24}\,12(n^2+2n+1)}{n^2}-14$$

$$=\lim_{n\to\infty}\cancel{20}\,12+\frac{24}{n}+\frac{12}{n^2}-14, \quad \frac{24}{n}\to 0, \quad \frac{12}{n^2}\to 0$$

$$=-2$$

2. (8 points) A particle on a line has velocity given by the function $v(t) = (2t-1)(t^2-t)$. On the interval [-2,0.5], find

(a) the displacement of the particle.

(b) the total distance traveled of the particle.

show work, give exact answer in either fract or dec.

(a) $\int_{-2}^{0.5} (2t-1)(t^2-t)\,dt$ ✓ $\to -3t^2$

$= \int_{-2}^{0.5} (2t^3 - t^2 - 2t^2 + t)\,dt$ ✓

$= \frac{2t^4}{4} - t^3 + \frac{t^2}{2} \Big|_{-2}^{0.5}$ ✓

$= \frac{1}{32} - \cancel{\frac{1}{8}} + \cancel{\frac{1}{8}} - (8+8+2)$ ✓

$= \frac{1}{32} - 18 = \boxed{-17.96875}$ $= \frac{-575}{32}$

$v(t) = 0$ @ $t=0$, $t = \frac{1}{2}$, $t = 1$

Allow 1 calc error.

(b) $\left| \int_{-2}^{0} (2t-1)(t^2-t)\,dt \right| + \left| \int_{0}^{0.5} (2t-1)(t^2-t)\,dt \right|$ ✓✓

$= \left| \frac{2t^4}{4} - t^3 + \frac{t^2}{2} \Big|_{-2}^{0} \right| + \left| \frac{2t^4}{4} - t^3 + \frac{t^2}{2} \Big|_{0}^{0.5} \right|$

$= |0 - (8+8+2)| + \left| \frac{1}{32} - \frac{1}{8} + \frac{1}{8} - 0 \right|$ ✓

$= 18 + \frac{1}{32} = \boxed{18.03125}$

−1 or −1.5 for splitting up in wrong place, no work.

only 1.5 if don't split up

Test A **No Calculator** Name: Key

3. (8 points) Evaluate the indefinite integrals:

(a) $\int \frac{-\csc^2(\ln x)}{x}\,dx$

$u = \ln x$

$du = \frac{1}{x}\,dx$

$x\,du = dx$

$= \int \frac{-\csc^2(u)}{\cancel{x}} \cdot \cancel{x}\,du$

$= \int -\csc^2(u)\,du$

$= \cot u + C$

$= \boxed{\cot(\ln x) + C}$

general idea

(b) $\int \left(x^{-1} + \sec(x)\tan(x) - \frac{1}{\sqrt{1-x^2}}\right)dx$

$= \boxed{\ln|x| + \sec x - \sin^{-1}(x) + C}$

or $+ \cos^{-1}(x)$

4. (4 points) Find $h'(x)$ if $h(x) = \int_{\pi}^{x^3} \sin(t)\cos(t)\,dt$. Show your work, and if you use the Fundamental Theorem of Calculus, you must identify where it is used. ⇐ or explain how it relates to the problem

Let $G(t)$ be an antiderivative for $\sin(t)\cos(t)$.

Then $\left.\begin{aligned}\int_{\pi}^{x^3} \sin(t)\cos(t)\,dt = G(t)\Big|_{\pi}^{x^3}\end{aligned}\right\} \to$ F.T.C. ✓

✓

$$h(x) = G(x^3) - G(\pi)$$

$$\Rightarrow h'(x) = G'(x^3)(3x^2) - 0$$

$$= \boxed{\sin(x^3)\cos(x^3)(3x^2)}$$ ✓✓

or

$h'(x) = \sin(x^3)\cos(x^3)(3x^2)$ ✓✓

Here I used the fund. Theorem of Calculus which says that if $h(u) = \int_{\pi}^{u} \sin t \cos t\, dt$, then $h'(u) = \sin u \cos u$. ✓✓

But $u = x^3$, so I have chain rule to get a $3x^2$ on the outside.

5. (4 points) Evaluate the definite integral

$$\int_0^2 (3x^3 - 7)\,dx$$

(Note: should get same answer as for question 1)

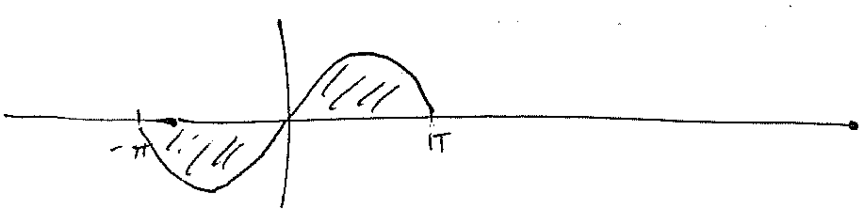
$$= \frac{3}{4}x^4 - 7x \Big|_0^2$$

$$= \frac{3}{4}(2^4) - 7(2) - (0 - 0)$$

$$= 12 - 14$$

$$= -2$$

Extra Credit(1 point) Sketch the graph of $\sin x$ and use the graph to explain how to find $\int_{-\pi}^{\pi} \sin x\,dx$ without taking an antiderivative.



Same area above x-axis as below

$$\Rightarrow \int_{-\pi}^{\pi} \sin x\,dx = \boxed{0}$$